

## STUDY GUIDE:

### Module 8: Rational Numbers, Part 5:

With this module we complete our treatment of rational numbers. We shall discuss the quotient of two decimal fractions. The key point is that when we divide two decimal fractions, we may move the decimal point *the same number of places (to the right)* without changing the value of the quotient. In this way we can convert the quotient of two decimal fractions into an equivalent problem that involves the quotient of two whole numbers.

We conclude the module with a study of what happens when we divide one decimal fraction by another and the quotient continues endlessly. Here the key point is that if we try to convert a common fraction in lowest terms to a decimal fraction, the resulting decimal will "terminate" if and only if the denominator has no prime factors other than 2 or 5. If other prime factors occur in the denominator, the resulting decimal will never "terminate" but will eventually repeat the same cycle of digits endlessly. What this really means is part of the study in this module.

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#### SPECIAL POINT:

*THIS MODULE ENDS OUR STUDY OF THE TECHNIQUES OF ARITHMETIC. FOR THE REST OF THE COURSE, EMPHASIS WILL BE ON PRACTICAL APPLICATIONS OF ARITHMETIC. IN DOING APPLICATIONS IT SAVES TIME AND ENERGY TO USE A CALCULATOR FOR DOING THE ARITHMETIC. BEFORE WE WANT TO TURN YOU LOOSE WITH THE CALCULATOR, WE WANT TO TEST YOUR RETENTION OF THE BASIC SKILLS. FOR THIS REASON MAKE SURE YOU DO THE "CUMULATIVE REVIEW" WHICH FOLLOWS THIS MODULE BEFORE YOU BEGIN WORK ON MODULE 9.*

ONCE YOU PASS THE "CUMULATIVE REVIEW" YOU MAY USE A CALCULATOR FOR THE REST OF THIS COURSE, INCLUDING THE FINAL EXAMINATION.

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Step 1:

View Videotape Lecture #8.

Step 2:

Read Module 8 of the text.

Step 3:

When you feel that you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

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Check The Main Ideas

In this module we learn how to \_\_\_\_\_ one *divide*  
decimal fraction by another. The basic idea is that  
we can \_\_\_\_\_ both divisor and dividend by the same *multiply*  
\_\_\_\_\_ whole number without changing the quotient. *non-zero*  
In particular, to multiply a decimal fraction by 10  
we move the decimal point one place to the \_\_\_\_\_. *right*  
So if we move the decimal point 2 places to the right  
we have multiplied the decimal fraction by \_\_\_\_\_. *100*  
In other words if we move the decimal point in both the  
divisor and dividend two places to the right we have  
multiplied both by \_\_\_\_\_ and hence we have not *100*  
changed the \_\_\_\_\_. *quotient*

Let's look at an example. Suppose we want to  
divide 1.84 by 0.46. To convert 1.84 to a whole  
number, we have to move the decimal point at least  
\_\_\_\_\_ places to the \_\_\_\_\_. 0.46 also becomes *two; right*  
a whole number if we move the decimal point two



places to the right. This tells us that

$1.84 \div 0.46$  is the same ratio as  $184 \div \underline{\hspace{1cm}}$ .

46

Since  $184 \div 46 = 4$ , we may conclude that

$1.84 \div 0.46 = \underline{\hspace{1cm}}$ . That is, 1.84 is  $\underline{\hspace{1cm}}$  times  
as great as 0.46.

4; 4

Sometimes the dividend and the divisor have

different numbers of decimal digits. For

example, suppose we want to divide 0.002 by

0.0004; that is  $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}}$ . To convert

0.002; 0.0004

0.002 to a whole number we must move the decimal

point at least  $\underline{\hspace{1cm}}$  places to the right; and to

3

convert 0.0004 to a whole number we must move the

decimal point at least  $\underline{\hspace{1cm}}$  places to the right.

4

But in order not to change the quotient we must

move the decimal point the  $\underline{\hspace{1cm}}$  number of places

same

in both 0.002 and 0.0004. Since 4 exceeds 3, to

convert  $0.002 \div 0.0004$  into an equivalent quotient

involving only whole numbers, we must move the

decimal point in both at least  $\underline{\hspace{1cm}}$  places to

4

the right. That is,  $0.002 \div 0.0004$  is equivalent

to  $\underline{\hspace{1cm}} \div 4$ . Since  $20 \div 4 = 5$ , we may conclude

20

that  $0.002 \div 0.0004 = \underline{\hspace{1cm}}$ . This is another way

5

of saying that as small as 0.002 is, it is still

$\underline{\hspace{1cm}}$  greater than 0.0004. That is, we must

5 times (Don't omit "times")

$\underline{\hspace{1cm}}$  0.0004 by 5 to get  $\underline{\hspace{1cm}}$  as the product.

multiply; 0.002

After we've converted the two decimal fractions to whole numbers, it is possible that the quotient will be a non-\_\_\_\_\_ decimal fraction. If, for example, the quotient is  $0.\overline{37}$ , we mean that the sequence of digits "\_\_\_\_" will repeat endlessly.

*terminating*

37

Since  $0.\overline{37}$  means 0.373737.... we can round it off to any desired degree of accuracy. For example to round  $0.\overline{37}$  off to the nearest thousandth, we look at the digit to the \_\_\_\_\_ of the thousandths digit. The thousandths-digit is \_\_\_\_\_ and the digit to its right is \_\_\_\_\_. Since 7 is greater than 5, we replace the thousandths-digit by \_\_\_\_\_ and drop the remaining digits to the right to obtain \_\_\_\_\_.

*right*

3

7

4

0.374

In other words in terms of multiples of a thousandth,  $0.\overline{37}$  is between 0.373 and \_\_\_\_\_ but closer to \_\_\_\_\_.

0.374; 0.374

In the language of inequalities we'd write:

$$\underline{\hspace{1cm}} < 0.\overline{37} < \underline{\hspace{1cm}}$$

0.373; 0.374

A quick way to remember how to read an inequality

symbol is that the arrow always points to the

\_\_\_\_\_ number. Thus both  $6 < 8$  and  $8 > 6$ .

*lesser (smaller)*

tell us that 6 is \_\_\_\_\_ than 8.

*less*

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#### Step 4:

Do the Mastery Review on the next page.



Mastery Review:ANSWERS:

1. The cost of a \$30 gift is to be shared equally by 8 people. How much should each person pay? 1. \_\_\_\_\_
2. A gift is \$40.74 and the cost is to be shared equally by 7 people. How much should each person pay? 2. \_\_\_\_\_
3. Write  $5 \div 16$  as a decimal fraction. 3. \_\_\_\_\_
4. (a) Write  $\frac{5}{8}$  as a decimal fraction. 4. (a) \_\_\_\_\_  
(b) Write  $\frac{10}{16}$  as a decimal fraction. (b) \_\_\_\_\_
5. Express  $\frac{4}{50}$  as a decimal fraction. 5. \_\_\_\_\_
6. Express  $\frac{2}{3}$  as an equivalent decimal fraction. 6. \_\_\_\_\_
7. What is the repeating cycle of digits when  $5 \div 11$  is written as a decimal fraction? 7. \_\_\_\_\_
8. Write  $11 \div 15$  as a decimal fraction. 8. \_\_\_\_\_
9. Write  $73 \div 99$  as a decimal fraction. 9. \_\_\_\_\_
10. Which decimal fraction names the greater ratio, 0.1003 or 0.0998? 10. \_\_\_\_\_
11. Round off  $0.\overline{37}$  to the nearest tenth. 11. \_\_\_\_\_
12. Round off  $0.\overline{37}$  to the nearest hundredth. 12. \_\_\_\_\_
13. Write  $0.003 \div 0.0005$  as a decimal fraction. 13. \_\_\_\_\_
14. Which is the greater ratio:  
 $0.037 \div 0.58$  or  $37 \div 580$  ? 14. \_\_\_\_\_
15. Write  $51.68 \div 1.7$  as a decimal fraction. 15. \_\_\_\_\_
16. What decimal fraction must we multiply by 1.7 to get 51.68 as the product? 16. \_\_\_\_\_
17. Write 0.7% as a common fraction in lowest terms. 17. \_\_\_\_\_
18. How much is 21.6% of 90,000? 18. \_\_\_\_\_
19. How much is  $32\frac{2}{7}\%$  of 28,000? 19. \_\_\_\_\_

Mastery Review: (cont)ANSWERS:

20. Write 0.8427 as the quotient of two whole numbers. 20. \_\_\_\_\_
21. Write  $0.\overline{31}$  as the quotient of two whole numbers. 21. \_\_\_\_\_
22. Which is correct,  $7 > 5$  or  $5 > 7$ ? 22. \_\_\_\_\_
23. Do  $8 > 3$  and  $3 < 8$  mean the same thing? 23. \_\_\_\_\_
24. Use the inequality symbols to indicate that 623 is between the 62nd and the 63rd multiple of 10. 24. \_\_\_\_\_
25. By locating 8.73 and 6.17 between consecutive whole numbers, use inequality symbols to express the range 8.73 X 6.17 is in. 25. \_\_\_\_\_

Answers to Mastery Review

1. \$3.75    2. \$5.82    3. 0.3125    4. (a) 0.625    (b) 0.625
5. 0.08    6.  $0.\overline{6}$     7. "45"    8.  $0.\overline{73}$     9.  $0.\overline{73}$
10. 0.1003    11. 0.4    12. 0.37    13. 6    14. They're equal
15. 30.4    16. 30.4    17.  $\frac{7}{1,000}$     18. 19,440
19. 9,040    20.  $8,427 \div 10,000$     21.  $31 \div 99$
22.  $7 > 5$     23. Yes    24.  $620 < 623 < 630$
25.  $48 < 8.73 \times 6.17 < 63$

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Step 5:

Do Self-Test 8, Form A on the next page.

Self-Test 8, Form A

ANSWERS:

1. Write each of the following as a decimal fraction:  
(a)  $0.012 \div (0.004 \div 0.0002)$   
(b)  $(0.012 \div 0.004) \div 0.0002$
2. Rounded off to the nearest tenth, how much is  $12.56 \div 3.4$ ?
3. To the nearest cent, how much is 0.07% of \$1,960?
4. Fill in the blank:  
(a) 30% of 90 is \_\_\_\_\_.  
(b) 30% of \_\_\_\_\_ is 90.
5. (a) Write  $0.\overline{783}$  as a common fraction in lowest terms.  
(b) Round off  $0.\overline{783}$  to the nearest millionth.  
(c) Write  $0.\overline{783}$  as a common fraction in lowest terms.
6. A gallon of paint covers 345 square feet. To the nearest tenth of a gallon, how much paint will it take to cover 1,670 square feet?
7. A salesperson earns 16% commission on all the clothes she sells. If her commission for selling clothes was \$268, how many dollars worth of clothes did she sell?
8. 17 people in an office decide to share equally the cost of a gift for an ill co-worker. If the gift costs \$125, how much must each pay? Round your answer up to the nearest dime.
9. You decide that the most you can afford each month for rent is 35% of your monthly take-home pay. If your monthly take-home pay is \$1,487.47, what is the most you can afford to pay for rent each month. Round your answer off to the nearest dollar.
10. Under the same condition as in exercise 9, what is the least your monthly take-home pay can be if your monthly rent is \$600. Again, round off your answer to the nearest dollar.

(ANSWERS ARE ON THE NEXT PAGE)



Answers for Self-Test 8, Form A

1. (a) 0.0006 (b) 15,000
2. 3.7
3. \$1.37
4. (a) 27 (b) 300
5. (a)  $\frac{29}{37}$  (b) 0.783784 (c)  $\frac{47}{60}$
6. 4.8 gallons
7. \$1,675
8. \$7.40
9. \$521
10. \$1,714

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If you did each problem in Self-Test 8, Form A correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 6.

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Step 6:

Study the solutions to Self-Test 8, Form A on the following pages, giving special emphasis to any problems you failed to answer correctly.

## Solutions for Self-Test 8, Form A

1.

Aside from trying to show that division of decimal fractions does not have the associative property, this exercise tries to show the importance of *relative size* when we divide decimal fractions. Notice that 0.012, 0.004, and 0.0002 are usually considered to be small numbers; yet in part (a) the quotient is quite small while in part (b) it is quite large.

(a)

Doing what's inside the parentheses first

we have:

$$0.004 \div 0.0002 =$$

$$40 \div 2 =$$

$$20$$

Hence:

$$0.012 \div (0.004 \div 0.0002) =$$

$$0.012 \div 20 =$$

$$\begin{array}{r} 0.0006 \\ 20 \overline{)0.0120} \end{array} \begin{array}{l} \text{+ (We annexed this 0 because} \\ \text{-120 } 120 \text{ is divisible by 20)} \end{array}$$

Note that we could have converted the numbers to equivalent common fractions. For example,

$$\begin{aligned} 0.012 \div 20 &= \frac{12}{1,000} \div 20 \\ &= \frac{12}{1,000} \times \frac{1}{20} = \\ &= \frac{12}{20,000} \\ &= \frac{6}{10,000} \\ &= 0.0006 \end{aligned}$$

We're moving the decimal points 4 places to the right to make both numbers whole numbers.

To avoid using decimals, we could have moved the decimal point in both numbers 3 places to the right to get  $12 \div 20,000$  which reduces to  $6 \div 10,000$  or 0.0006.

This could be reduced to  $3/5,000$  but 10,000 is a better denominator than 5,000 when we want to convert rational numbers to decimal fractions.

Solutions for Self-Test 8, Form A (cont)

1 (b).

(b)

$$0.012 \div 0.004 =$$

$$12 \div 4 =$$

3

Hence:

$$(0.012 \div 0.004) \div 0.0002 =$$

$$3 \div 0.0002 =$$

$$30,000 \div 2 =$$

15,000

Note that if the parentheses are omitted in parts (a) and (b) both problems look alike; yet the grouping changes an answer from 0.0006 to 15,000.

2.

As indicated in the text, the key to dividing two decimal fractions is to have the divisor in the form of a whole number. Since the divisor in this exercise is 3.4 we want to move the decimal point 1 place to the right. To keep the ratio the same, we must also move the decimal point 1 place to the right in the dividend (12.56). So we have:

$$12.56 \div 3.4 =$$

$$\begin{array}{r} 3.69 \approx 3.7 \text{ (that is, since} \\ 125.6 \div 34 = 34 \overline{)125.60} \text{ the digit in the} \\ \underline{-102} \text{ hundredths-place} \\ 236 \text{ is more than 5, we} \\ \underline{-204} \text{ replace the 6 in} \\ 320 \text{ 3.69 to 7 and get} \\ \underline{-306} \text{ 3.70 or 3.7)} \\ 14 \end{array}$$

We're moving the decimal point 3 places to get whole numbers.

We're moving the decimal point 4 places to the right to convert 0.0002 into 2. To keep the ratio the same, we're moving the decimal point 4 places to the right in 3(.) to get 30000(.)

We could continue to annex 0's but since we're rounding off to the nearest tenth, we do not have to go beyond the hundredths-place.



## Solutions for Self-Test 8, Form A (cont)

### 2. (cont)

#### Some Notes on Exercise 2:

(1) We could have moved the decimal point 2 places in both dividend and divisor to get the whole number problem:  $1,256 \div 340$ . This eliminates the appearance of decimal fractions, but it gives us greater numbers to deal with.

(2) Save the rounding off to the last step. That is, do the division first and then round off. Do not round off the dividend and divisor first. For example, if we round off 12.56 to the nearest tenth we get 12.6.

$$12.6 \div 3.4 =$$

$$\begin{array}{r} 126 \div 34 = 34 \overline{)126.00} \\ \underline{-102} \phantom{00} \\ 240 \\ \underline{-238} \phantom{00} \\ 20 \\ \underline{-00} \phantom{00} \end{array}$$

(3) If you don't mind working with larger numbers, there is no need to reduce ratios to lowest terms.

For example had we replaced  $126 \div 34$  by  $63 \div 17$  we'd be using smaller numbers in finding the quotient, but the process and the answer would be the same.

### 3.

The most common error in this exercise is to forget to take the percent sign into account. Remember that 0.07% means  $0.07 \div 100$ ; and in decimal form we divide by 100 by moving the decimal point

Remember that we are free to move a decimal point as many places as we wish. We just have to make sure that whatever we do to the dividend, we must also do to the divisor.

It happens here that to the nearest tenth we still get the same answer, but the quotient isn't the same as before (3.69...). Sometimes the answer will be different as illustrated in Exercise 9.

That is, 126 and 34 share 2 as a common factor.

Solutions for Self-Test 8, Form A (cont)

3. (cont)

2 places to the right. That is:

$$0.07\% = 0.0007.$$

"of" still means "X".

Therefore:

$$0.07\% \text{ of } \$1,960 =$$

$$0.0007 \times \$1,960 =$$

$$\$1.3720$$

Since we want the nearest cent, we look at the digit in the thousandths-place (2) and since it's less than 5 we drop it and keep the answer as \$1.37

Note that if you did the problem as  $0.07 \times \$1,960$  and got \$137.20 as the answer, you forgot about the percent sign. In fact what you found was 7% of \$1,960--not 0.07% of \$1,960.

The point is that 0.07% means 0.07 per 100 or 7 per 10,000; and at this rate you're taking less than \$2 per \$2,000.

4.

The basic aim of this exercise is to make sure that you catch the subtle difference between multiplication and division. In many word-problems it is easy to confuse the two.

$7 \times \$1,960 = \$13,720$  but we have to move the decimal point 4 places to the left because 0.0007 has 4 digits to the right of the decimal point.

There is a quick way to get a rough check. Namely 0.07% is less than 1% and 1% of \$1,960 is \$19.60. So the answer has to be less than \$19.60--which \$137.20 isn't!

In fact if we divide both 7 and 10,000 by 5 we get 7/5 per 2,000 or \$1.40 per \$2,000. Based on this, \$1.37 per \$1,960 is quite logical.

Exercise 7 treats this idea in more detail. So also do Exercises 9 and 10.



Solutions for Self-Test 8, Form A (cont)

4. (cont)

(a)

$$30\% \text{ of } 90 =$$

$$0.30 \times 90 =$$

$$27$$

$$\text{In terms of common fractions, } 30\% = \frac{30}{100} = \frac{3}{10}.$$

$$\frac{3}{10} = \frac{3 \times 9}{10 \times 9} = \frac{27}{90}$$

(b)

Using the language of decimal fractions for the  
extra practice,  $30\% = 30 \div 100$

$$= 0.30$$

$$= 0.3$$

Hence:

$$30\% \text{ of } \underline{\hspace{2cm}} = 90$$

means

$$0.3 \times \underline{\hspace{2cm}} = 90; \text{ or}$$

$$\underline{\hspace{2cm}} = 90 \div 0.3$$

$$= 900 \div 3$$

$$= 300$$

$$\text{Check: } 30\% \text{ of } 300 =$$

$$0.3 \times 300 =$$

$$90.0 =$$

$$90$$

Alternative Method:

If 30% of a number is 90, then 1% is  $90 \div 30$   
or 3. And if 1% is 3, then 100% is  $3 \times 100$  or 300.

In summary, notice the big difference between  
parts (a) and (b) even though they might seem alike  
when you read them quickly.

Again, don't be afraid to  
estimate. 30% means 30 per  
100. At that rate, we're  
taking a "bit" less than 30  
per 90.

Remember that 0.30 and 0.3  
mean the same thing. Either  
form tells us that we have  
3 tenths and no hundredths.

Don't let the decimal points  
confuse you with this con-  
cept. For example, if we  
had  $3 \times \underline{\hspace{1cm}} 900$ , we should,  
from Module 3, recognize it  
as  $\underline{\hspace{1cm}} = 900 \div 3$ .

Think of 30 as the adjective  
and "%" as the noun. If 30  
"percents" equal 90, then  
each "percent" is  $1/30$  of 90  
or 3. Hence 100 "percents"  
is 100 three's or 300.



Solutions for Self-Test 8, Form A (cont)

5.

We've already seen that when a rational number is expressed as a decimal fraction, the fraction will either eventually end or else repeat the same cycle of digits endlessly. In this exercise we want to emphasize the procedure for converting a repeating decimal fraction into common fractions.

(a)

The "bar" over 783 tells us that the repeating cycle of digits is "783". Since the repeating cycle consists of 3 digits, we get the same cycle of decimal digits every time we move the decimal point 3 places to the right. Moving the decimal point 3 places to the right multiplies the number by 1,000.

If we call the number  $N$ , we have:

$$\begin{aligned} N &= 0.\overline{783} \\ &= 0.783783783783783\dots \end{aligned}$$

$$\begin{array}{r} 1,000 N's = 783.\overline{783783783} \dots \\ - 1 N = 0.\overline{783783783} \dots \\ \hline 999 N's = 783 \end{array}$$

But 999  $N$ 's is the same as writing

999  $\times N$ . So we've shown that:

$$999 \times N = 783$$

Therefore:

$$\begin{aligned} N &= 783 \div 999 \\ &= \frac{783}{999} \end{aligned}$$

*We may, for the most part, even view a terminating decimal fraction as a special case of a repeating decimal. Namely, it repeats "0" endlessly.*

*This step is to emphasize that "783" is the repeating cycle.*

*See the strategy? Since the decimal fractions parts are the same, they "cancel" when we subtract.*

*This is our adjective-noun theme again. 1,000  $N$ 's - 1  $N$  is 999  $N$ 's because 1,000 - 1 is 999.*

*Again be careful.*

*999  $\times N = 783$  looks like a multiplication problem, but it's really a division problem.*

# Solutions for Self-Test 8, Form A (cont)

5 (a). (cont)

The only problem with  $\frac{783}{999}$  is that it's not in lowest terms. But as indicated from our margin note, we can rewrite it as:

$$\begin{aligned}\frac{783}{999} &= \frac{3 \times 3 \times 3 \times 29}{3 \times 3 \times 3 \times 37} \\ &= \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times 29}{\cancel{3} \times \cancel{3} \times \cancel{3} \times 37} \\ &= \frac{29}{37}\end{aligned}$$

Check:

$$\begin{array}{r} 29 \overline{) 29.000} \\ \underline{-25 \ 9} \phantom{00} \\ 3 \ 10 \phantom{00} \\ \underline{-2 \ 96} \phantom{00} \\ 140 \phantom{00} \\ \underline{-111} \phantom{00} \\ 29 \phantom{00} \end{array}$$

(see why this cycle will repeat endlessly? When we started we had 29 as a remainder. That is, our dividend was 29 followed by endlessly many 0's; and after our last subtraction we have 29 followed by endlessly many 0's. So the cycle 783 will keep repeating.)

(b)

The main purpose of this part of the exercise is to make sure you don't confuse  $0.\overline{783}$  with 0.783. To round  $0.\overline{783}$  to the nearest millionth, we have to look at the digit in the ten-millionths place.

That is:

$$0.\overline{783} = 0.7837837$$

↑

Since this digit is more than 5, we take the digit to its left (3) and increase it by 1 to get 0.783784.

More specifically,  $0.\overline{783}$  is between the two terminating decimal fractions 0.783783 and 0.783784; but closer in value to 0.783784.

$$\begin{array}{r} 29 \phantom{00} \\ 3 \overline{) 87} \\ \underline{3 \ 261} \\ 3 \overline{) 783} \end{array} \qquad \begin{array}{r} 37 \phantom{00} \\ 3 \overline{) 111} \\ \underline{3 \ 333} \\ 3 \overline{) 999} \end{array}$$

29 and 37 are prime numbers so the factorization is complete.

It is easy to see that 999 is divisible by 3 as are 333 and 111. So it is easy to factor 999. Once that's done we only have to see whether 783 shares any of these factors. This motivates us to see if 783 is a multiple of 3.

With 37 as the denominator, we're sure that a remainder has to repeat by the 37th decimal place. In this problem it happened much sooner.

That is,  $0.\overline{783}$  is  $29/37$  and 0.783 is  $783/1,000$ .  $29/37$  and  $783/1,000$  are not equivalent.

The idea is that we want 6 decimal place accuracy in this problem, so we have to look at the 7th decimal digit in order to round off properly.

Compare this with 0.783. To the nearest millionth, 0.783 is 0.783000. So 0.783 is quite different from 0.783.



Solutions for Self-Test 8, Form A (cont)

5(c)

Here we have to pay close attention to the placement of the bar. It is only over the 3. Hence "78" is not part of the repeating cycle. That is, if we call the number M, we have:

$$M = 0.78\bar{3} = 0.783333\bar{3}...$$

Since the repeating part of the decimal is just the single digit 3, the first place we can put the decimal point to get this cycle is between the 8 and the 3. That is, we have to move the decimal point 2 places to the right, which is the same as multiplying by 100. Hence:

$$100 \times M = 78.3333\bar{3}.... \quad (1)$$

If we move the decimal point one more place to the right, we get the same repeating cycle.

Namely:

$$1,000 \times M = 783.333\bar{3}... \quad (2)$$

Combining (1) and (2) and subtracting, we get:

$$\begin{array}{r} 1,000 M's = 783.333\bar{3}.... \\ - 100 M's = 78.3333\bar{3}... \\ \hline 900 M's = 705.0000 \end{array}$$

or

$$900 \times M = 705$$

so:

$$M = 705 \div 900$$

$$= \frac{705}{900}$$

$$= \frac{3 \times 5 \times 47}{3 \times 5 \times 60}$$

*We used N in part (a). Since this is a different number, we name it by a different letter of the alphabet.*

*To get (1) we've already moved the decimal point of M 2 places to the right. Moving another place to the right then moves it a total of 3 places to the right in M.*

*Our strategy is get a form in which the two numbers we are subtracting have exactly the same repeating decimal fraction.*

$$\begin{array}{r} 47 \\ 3 \overline{)141} \\ 5 \overline{)705} \end{array} \qquad \begin{array}{r} 60 \\ 3 \overline{)180} \\ 5 \overline{)900} \end{array}$$

*Since 47 and 60 have no common prime factors we do not have to go further in reducing 705/900 to lowest terms.*



Solutions for Self-Test 8, Form A (cont)

5(c) (cont)

Therefore, after cancelling we see that

$$M = \frac{47}{60}$$

Comparing parts (a) and (c), we see that

the difference between  $0.\overline{783}$  and  $0.78\overline{3}$  is the same as the difference between  $\frac{29}{37}$  and  $\frac{47}{60}$ .

More specifically:

$$\frac{29}{37} = \frac{29 \times 60}{37 \times 60} = \frac{1,740}{2,220}$$

$$\frac{47}{60} = \frac{47 \times 37}{60 \times 37} = \frac{1,739}{2,200}$$

So the difference between  $0.\overline{783}$  and  $0.78\overline{3}$

is 1 part per 2,220 parts.

6.

Each gallon of paint covers 345 square feet.

So to see how many gallons of paint are needed to cover 1,670 square feet, we have to see what multiple of 345 equals 1,670. To do this we divide 1,670 by 345.

$$\begin{array}{r} 4.84 \div 4.8 \\ 1,670 \div 345 = 345 \overline{)1,670.00} \\ \underline{-1,380} \phantom{00} \\ 290 \phantom{0} \\ \underline{-276 \phantom{0}} \\ 14 \phantom{00} \\ \underline{-13 \phantom{80}} \\ 20 \end{array}$$

We want the answer to the nearest tenth of a gallon, so we can stop the division process after we know the digit in the hundredths-place. Since this digit (4) is less than 5, we drop it and write the answer as 4.8 gallons.

Check:  $\frac{47}{60} \overline{)0.7833...}$

$$\begin{array}{r} 0.7833... \\ \underline{-42 \phantom{00}} \\ 5 \phantom{00} \\ \underline{-4 \phantom{80}} \\ 200 \\ \underline{-180} \\ 20 \end{array}$$

(20 is the repeating remainder.)

Don't become a slave to memorizing a technique. Use your common sense or whatever seems natural to you. For example:

1 gal. covers 345 sq. ft.  
2 gal. covers 690 sq. ft.  
3 gal. covers 1,035 sq. ft.  
4 gal. covers 1,380 sq. ft.  
5 gal. covers 1,725 sq. ft.  
(the sq. ft. are multiples of 345). Since 1,670 is between 1,380 and 1,725 we know that we need between 4 and 5 gallons of paint.

Again notice that we could have replaced  $1,670 \div 345$  by  $334 \div 69$ ; but it's not clear if it's worth the effort.

Solutions for Self-Test 8, Form A (cont)

6. (cont)

As a check we notice that with 4 gallons we could cover 1,380 square feet and with 5 gallons we could cover 1,725 square feet. Since 1,670 square feet is much closer to 1,725 square feet than to 1,380 square feet, we expect the answer to be less than but close to 5 gallons. So 4.8 gallons is a feasible answer.

*In most real-life situations we'd buy 5 gallons. The little that's left over can be a safety factor in case we have to do something over.*

Note:

"square feet" is a measure of area which we study in Module 11. So you aren't expected to know about it at this point of the course. If you aren't comfortable with the concept of area, you should still be able to do this problem. After all, we may view "square feet" simply as a noun that is modified by 345.

*It's like saying that 1 oog equals 345 coogs. You don't have to know what "oog" and "coog" mean to see that at this rate 2 oogs will equal 690 coogs and so on.*

7.

Once we read this problem correctly, the arithmetic is the same as that used in Exercise 4, part (b). Notice that to say that the salesperson earns a 16% commission on all clothes she sells means mathematically that:

$$16\% \text{ of clothing sales} = \text{commission earned} \quad (1)$$

Now comes the reading comprehension! Notice that \$268 is the commission earned and not the amount of the sales. That is, the \$268 replaces "commission earned" in (1).

Hence, to find the total clothing sales we rewrite (1) as:

$$16\% \text{ of clothing sales} = \$268 \quad (2)$$

*If you did the problem incorrectly by computing 16% of \$268, you were solving the wrong problem. 16% of \$268 would tell you the commission if the total clothing sales were \$268. This isn't what the problem asks for.*



Solutions for Self-Test 8, Form A (cont)

7. (cont)

Replacing 16% by 0.16; "of" by "X";  
and "clothing sales" by a "blank", we get:

$$0.16 \times \text{_____} = \$268$$

Mathematically, this is the same as:

$$\begin{aligned} \text{_____} &= \$268 \div 0.16 \\ &= \$26,800 \div 16 \\ &\quad \underline{\$1,675} \\ &= 16 \overline{) \$26,800} \\ &\quad \underline{-16} \\ &\quad 108 \\ &\quad \underline{-96} \\ &\quad 120 \\ &\quad \underline{-112} \\ &\quad 80 \\ &\quad \underline{-80} \\ &\quad 0 \end{aligned}$$

Check:

$$16\% \text{ of } \$1,675 = 0.16 \times \$1,675 = \$268$$

A Note On Common Fractions

$$16\% \text{ means } \frac{16}{100} = \frac{16 \times 20}{100 \times 20} = \frac{320}{2,000}$$

This tells us that her commission is \$320 per \$2,000 of sales. So \$268 commission per \$1,675 of sales seems plausible.

8.

The fact that the denomination of our answer is "dollars per person" tells us to divide "dollars" by "people". So in this case we have that each person has to pay  $\$(125 \div 17)$ .

So we have:

Again, use your common sense. The salesperson only gets a part of what she sells. So if her commission is \$268, she must have sold more than that amount of clothes. In other words, the answer to this problem has to be more than \$268, not less than \$268. This is another way of realizing that we have to divide by 0.16 not multiply by it.

$$\begin{array}{r} 1,675 \\ \times 16 \\ \hline 10,050 \\ 1675 \\ \hline 26,800 \end{array}$$

In fact, she gets, \$160 per \$1,000; \$320 per \$2,000; and \$480 per \$3,000. This also shows us that \$1,675 is in the right range.

$$\begin{aligned} \$125 \div 10 &= \$12.50 \\ \$125 \div 20 &= \$6.25 \end{aligned}$$

From the above we can see that with 17 people the cost per person is between \$6.25 and \$12.50 but closer to \$6.25 (because 17 is closer to 20 than to 10). So again we should be able to estimate the correct answer.



Solutions for Self-Test 8, Form A (cont)

8. (cont)

$$\begin{array}{r} 7.352 \\ 17 \overline{)125.000} \\ \underline{-119} \phantom{00} \\ 60 \phantom{00} \\ \underline{-51} \phantom{00} \\ 90 \phantom{00} \\ \underline{-85} \phantom{00} \\ 50 \phantom{00} \\ \underline{-34} \phantom{00} \\ 16 \end{array}$$

We can annex as many 0's as we want. The actual number we need depends on the degree of accuracy we require.

Notice that in this problem we did not say round off. Instead we said to round up. This is because it is less embarrassing to have money left over than to not have enough.

Most likely we'd have asked for \$8 from each person. That would give us  $\$8 \times 17$  or \$136. We could then buy a nice card, flowers etc. to supplement the gift.

At \$7.30 they raise  $\$7.30 \times 17$  or \$124.10.

At \$7.40 they raise  $\$7.40 \times 17$  or \$125.80

Notice that all that prevented us from doing this type of problem back in Module 3 is that we hadn't yet learned to work with fractional parts. In this sense, the basic ideas remain the same as we proceed through the course; only our computational ability increases.

9.

Exercises 9 and 10 are a matched pair that are meant to illustrate the difference mentioned in parts (a) and (b) of Exercise 4 and further discussed in Exercise 7. The key point is to realize what we're saying in this problem. Namely, the most we can pay for rent is 35% of our take-home pay. In terms of a "recipe":

Solutions for Self-Test 8, Form A (cont)

9. (cont)

35% of monthly take-home pay = maximum monthly rent.

If we let T stand for "Take-home pay" and

R for "Rent", the recipe becomes:

$$35\% \text{ of } T = R \quad (1)$$

In this context, the only difference between Exercises 9 and 10 is that in Exercise 9 we are given the value of T and asked to find the value of R. In Exercise 10 we are given the value of R and asked to find the value of T.

Getting back to Exercise 9, then, we are given our monthly take home pay (T). So we replace T by this value in (1) and solve for R.

That is:

$$\begin{aligned} & 35\% \text{ of } \$1,487.47 = R \\ \text{or} \quad & 0.35 \times \$1,487.47 = R \\ \text{or} \quad & \$520.6145 = R \end{aligned}$$

So rounded off to the nearest dollar, the most we can afford for rent each month is \$521.

Note:

Don't round off until the end of the problem.

For example, if you began by rounding \$1,487.47 off to \$1,487, the answer you'd get is:

$$0.35 \times \$1,487 = \$520.45 \approx \$520$$

It isn't a glaring difference, but notice that you've "lost" a dollar by rounding off too soon.

*This notation is a preview of what we'll be using in rest of the modules. When more than one blank is needed it is usually less confusing to use letters such as T and R in place of the blanks.*

*But it's not quite that simple because you are not told that. Part of the problem is to figure that out.*

*Again, don't be afraid to think in terms of whole numbers. We're saying that our maximum rent is \$35 per \$100 of take-home pay. This is the same rate as \$350 per \$1,000 and \$700 per \$2,000. Since \$1,487.47 is about half way between \$1,000 and \$2,000, the answer should be about half way between \$350 and \$700 (\$525 is half way between \$350 and \$700)*

Solutions for Self-Test 8, Form A (cont)

10.

In a way this is really another part of Exercise 9. We've separated into two problems to emphasize the difference. In this problem the given amount (\$600) represents the maximum rent per month. That is, in the formula:

$$35\% \text{ of } T = R$$

it is R that is replaced by \$600.

So we get:

$$35\% \text{ of } T = \$600$$

or

$$0.35 \times T = \$600$$

or

$$T = \$600 \div 35$$

$$= \$60,000 \div 35$$

$$\$12,000 \div 7 = 7) \$12,000.00$$

$$\begin{array}{r} \$1,714.28 \\ -7 \\ \hline 50 \\ -49 \\ \hline 10 \\ -7 \\ \hline 30 \\ -28 \\ \hline 20 \\ -14 \\ \hline 60 \\ -56 \\ \hline 4 \end{array}$$

Check:  $35\% \text{ of } \$1,714 = 0.35 \times \$1,714 = \$599.90$

$$35\% \text{ of } \$1,715 = 0.35 \times \$1,715 = \$600.25$$

Hint:

If you still get confused about knowing when to multiply and when to divide. Notice that the rent is only part of the take home pay. That is, the take-home pay has to be more than the rent and this won't happen if you do the wrong operation.

Recall from Exercise 9 that T denotes the monthly take-home pay and R denotes the maximum monthly rent.

35 and 600 are divisible by 5. Reducing to lowest terms gives us a single-digit divisor so it may be worth the effort.

Once we get to here we can see that to the nearest dollar the answer will be \$1,714.

$\begin{array}{r} 1714 \\ \times 35 \\ \hline 8570 \\ 5142 \\ \hline 59990 \end{array}$	$\begin{array}{r} 1715 \\ \times 35 \\ \hline 8575 \\ 5145 \\ \hline 60025 \end{array}$
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Solutions for Self-Test 8, Form A (concluded)

10. (cont.)

Some Concluding Notes on Exercise 10

(1) If you got the wrong answer by computing 35% of \$600, you were doing an Exercise 9 type problem by mistake. That is, you were letting  $T = \$600$  and as a result you found how much rent you could afford if your monthly take-home pay was \$600. This is not what was asked in the Exercise.

(2) Keep thinking in terms of whole numbers. If 35% is .600 then 1% is  $600 : 35 = 17^+$ . Hence 100% or the whole is  $17^+ \times 100$  or about 1,700.

More specifically:

$$\begin{array}{r} 35 \overline{)600} = 7 \overline{)120} = 17 \frac{1}{7} \\ \underline{-7} \phantom{0} \\ 50 \\ \underline{-49} \\ 1 \end{array}$$

So the "bottom line" is this. It isn't too helpful if you know how to do the various "recipes" of arithmetic or if you can use a calculator unless you can translate a problem from words into a mathematical relationship. For most people, this is the hardest part of the subject. But it is important that we keep stressing problem-solving. The most important step is to feel comfortable using whole numbers to get reasonable estimates.

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Step 7:

Do Self-Test 8, Form B on the next page.

1. Write each of the following as a decimal fraction:
  - (a)  $(0.072 \div 0.06) \div 0.003$
  - (b)  $0.072 \div (0.06 \div 0.003)$
2. Rounded off to the nearest tenth, how much is  $26.76 \div 6.3$ ?
3. To the nearest cent, how much is 0.09% of \$2,540?
4. Fill in the blank:
  - (a) 40% of 80 = \_\_\_\_\_.
  - (b) 40% of \_\_\_\_\_ = 80.
5.
  - (a) Write  $0.\overline{567}$  as a common fraction in lowest terms.
  - (b) Round off  $0.\overline{567}$  to the nearest millionth.
  - (c) Write  $0.5\overline{67}$  as a common fraction in lowest terms.
6. A gallon of paint covers 427 square feet. To the nearest tenth of a gallon, how much paint will it take to cover 2,380 square feet?
7. A salesperson earns 17% commission on all the clothes she sells. If her commission for selling clothes was \$391, how many dollars worth of clothes did she sell?
8. 19 people in an office decide to share equally the cost of a gift for a co-worker. If the gift costs \$160, how much must each pay? Round your answer up to the nearest dime.
9. You decide that the most you can afford each month for rent is 32% of your monthly take-home pay. If your monthly take-home pay is \$1,520.49, what is the most you can afford to pay for rent each month? Round your answer off to the nearest dollar.
10. Under the same condition as in Exercise 9, what is the least your monthly take-home pay can be if your monthly rent is \$525? Round off your answer to the nearest dollar.

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 8, Form B

1. (a) 400      (b) 0.0036
2. 4.2
3. \$2.29
4. (a) 32      (b) 200
5. (a)  $\frac{21}{37}$       (b) 0.567568      (c)  $\frac{511}{900}$
6. 5.6 gallons
7. \$2,300
8. \$8.50
9. \$487
10. \$1,641

\*\*\*\*\*

If you did each problem in Self-Test 8, Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

\*\*\*\*\*

Step 8:

View the solutions for Self-Test 8, Form B on Videotape Lecture 8S.

Pay special attention to the solutions of those problems for which you failed to get the correct answers.

Step 9:

Do Self-Test 8, Form C on the next page.



1. Write each of the following as a decimal fraction:  
(a)  $(0.06 \div 0.0012) \div 0.0004$   
(b)  $0.06 \div (0.0012 \div 0.0004)$
2. Rounded off to the nearest tenth, how much is  $23.89 \div 5.9$ ?
3. To the nearest cent, how much is 0.03% of \$3,790?
4. Fill in the blank:  
(a) 60% of 180 = \_\_\_\_\_.  
(b) 60% of \_\_\_\_ = 180.
5. (a) Write  $0.\overline{513}$  as a common fraction in lowest terms.  
(b) Round off  $0.\overline{513}$  to the nearest millionth.  
(c) Write  $0.\overline{513}$  as a common fraction in lowest terms.
6. A gallon of paint covers 565 square feet. To the nearest tenth of a gallon, how much paint will it take to cover 3,324 square feet?
7. A salesperson earns 14% commission on all the clothes she sells. If her commission for selling clothes was \$308, how many dollars worth of clothes did she sell?
8. 21 people in an office decide to share equally the cost of a gift for a co-worker. If the gift costs \$180, how much must each pay? Round your answer up to the nearest dime.
9. You decide that the most you can afford each month for rent is 33% of your monthly take-home pay. If your monthly take-home pay is \$1,637.42, what is the most you can afford to pay for rent each month? Round your answer off to the nearest dollar.
10. Under the same conditions as in Exercise 9, what is the least your monthly take-home pay can be if your monthly rent is \$575? Again, round off your answer to the nearest dollar.

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 8, Form C

1. (a) 125,000 (b) 0.02
2. 4.0 (or simply 4)
3. \$1.14
4. (a) 108 (b) 300
5. (a)  $\frac{19}{37}$  (b) 0.513514 (c)  $\frac{77}{150}$
6. 5.9 gallons
7. \$2,200
8. \$8.60
9. \$540
10. \$1,742

\*\*\*\*\*

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #8.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #9.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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REMINDER

Before you begin doing Module 9 in the textbook, make sure that you study the "CUMULATIVE REVIEW" that precedes the subject matter of Module 9.

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## STUDY GUIDE:

### Prelude to Module 9

Answers for

Despite the availability and low cost of hand-calculators and computers,<sup>1</sup> there is still much need to develop a number sense. Ultimately, there is no substitute for having a "feeling" about how to solve a problem and to know what the right answer means. We are now about to start that part of our course that is devoted to problem solving.

For this reason it is very important that you "check out" your arithmetic skills before continuing with the course. The "CUMULATIVE REVIEW" that follows is a self-quiz by which you may take an inventory of the skills you've developed. You'll notice that after each problem is a pair of numbers. The first tells you the Self-Test from which the problem is taken; and the second gives you the number of the problem in that Self-Test. For example, (7,3) would mean the 3rd exercise in Self-Test 7.

If you do the problem correctly, you will not need to take advantage of this code. However, should you get a problem wrong, look up the problem on Form A of the appropriate Self-Test. It will be the same problem as that on the "CUMULATIVE REVIEW" except with different numbers. Study the solution to the exercise as given in the Study Guide. When you feel you see what you did wrong, redo the problem in the "CUMULATIVE REVIEW".

After you've reviewed each problem and feel confident that you understand how to do them, continue with the study of Module 9.



CUMULATIVE REVIEW:ANSWERS:

1. Read the number represented by the numeral:

54,904,045,000,000,000

(1,8)

2. Write as a place value numeral: seventy two quadrillion four hundred sixty three billion.

(1,9)

3. By rounding off each term to the nearest trillion, estimate the sum:

5,983,345,089,772

6,098,432,987,543

2,973,334,089,512

4,234,678,987,345

(2,5)

4. (a) Write 34,034 - 9,876 as a place value numeral.

(b) Subtract 9,876 from 34,034.

(c) What must we add to 9,876 to get 34,034 as the sum?

(2,6), (2,7), and (2,8)

5. (a) Find the product of 534 and 700.

(b) How much is 534 X 699?

(3,4)

6. (a) How much is 15,432 ÷ 643?

(b) What number must we multiply by 643 to get 15,432 as the product?

(c) Is 15,432 a multiple of 643?

(3,7)

7. (a) How much is  $\frac{2}{7} + \frac{1}{4}$ ?

(b) You spend  $\frac{2}{7}$  of your salary for rent and  $\frac{1}{4}$  for food. If your salary is \$280, how much do you spend for rent and food?

(4,4)

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

5. (a) \_\_\_\_\_

(b) \_\_\_\_\_

6. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

7. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(continued)

CUMULATIVE REVIEW: (cont)

ANSWERS:

8. Write 69,300 as a product of prime numbers.

(4,6)

9. (a) Find the least common multiple of 30, 12, and 18.

- (b) Write  $\frac{1}{30} + \frac{5}{12} + \frac{7}{18}$  as a common fraction in lowest terms.

- (c) Express  $\frac{1}{30} - (\frac{5}{12} - \frac{7}{18})$  as a common fraction in lowest terms.

(4,7) and (4,8)

10. (a) How much is  $\frac{5}{13} \times \frac{3}{4}$ ?

- (b) What must we multiply by  $\frac{5}{13}$  to get  $\frac{3}{4}$ ?

(5,3)

11. (a) How much is  $20 \div \frac{1}{5}$ ?

- (b) A map uses a scale of  $\frac{1}{5}$  of an inch to represent 20 feet. How many feet is represented by:

(i) 1 inch? (ii)  $\frac{1}{4}$  of an inch?

(5,10)

12. Write as a single mixed number:

$$3\frac{1}{6} + 4\frac{2}{5} + 2\frac{1}{2}$$

(6,1)

13. Write as a single mixed number:

$$3\frac{1}{6} \times 4\frac{2}{5} \times 2\frac{1}{2}$$

(6,2)

14. Express 27% of  $8\frac{1}{6}$  as a mixed number.

(6,5)

15. Express  $\frac{4}{9}\%$  of 2,500 as a mixed number.

(6,6)

8. \_\_\_\_\_

9. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

10. (a) \_\_\_\_\_

(b) \_\_\_\_\_

11. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(i) \_\_\_\_\_

(ii) \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

(cont)

CUMULATIVE REVIEW: (concluded)

ANSWERS:

16. Compute:  $0.0809 + 0.99287 + 0.0057$

(7,1)

17. Compute: (a)  $(8.32 - 4.983) - 2.19$

(b)  $8.32 - (4.983 - 2.19)$

(7,2)

18. Compute: (a)  $7.32 \times 4.09 \times 0.09$

(b)  $(7.32 \times 4.09) + 5.91$

(c)  $7.32 \times (4.09 + 5.91)$

(7,3)

19. To the nearest cent, how much is  
0.09% of \$2,378?

(8,3)

20. Fill in the blank:

(a) 20% of 80 = \_\_\_\_\_.

(b) 20% of \_\_\_\_\_ = 80.

(8,4)

16. \_\_\_\_\_

17. (a) \_\_\_\_\_

(b) \_\_\_\_\_

18. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

19. \_\_\_\_\_

20. (a) \_\_\_\_\_

(b) \_\_\_\_\_

\*\*\*\*\*

Check your answers with those given on the next page.

If you got an incorrect answer, look up the corresponding problem on the appropriate Self-Test and study the solution. Then see if you can correct your mistakes.

\*\*\*\*\*

When you feel comfortable with your mastery of the computational skills presented in the CUMULATIVE REVIEW, continue with Module 9.

\*\*\*\*\*



ANSWERS TO THE CUMULATIVE REVIEW PROBLEMS:

1. 54 quadrillion 904 trillion 45 billion
2. 72,000,463,000,000,000
3. 19 trillion (19,000,000,000,000)
4. (a) 24,158 (b) 24,158 (c) 24,158
5. (a) 373,800 (b) 373,266
6. (a) 24 (b) 24 (c) yes
7. (a)  $\frac{15}{28}$  (b) \$150
8.  $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 11$  or  $2^3 \times 3^2 \times 5^3 \times 7 \times 11$
9. (a) 180 (b)  $\frac{151}{180}$  (c)  $\frac{1}{180}$
10. (a)  $\frac{15}{52}$  (b)  $\frac{39}{20}$
11. (a) 100 (b) (i) 100 feet (c) 25 feet
12.  $10\frac{1}{15}$
13.  $34\frac{5}{6}$
14.  $2\frac{41}{200}$
15.  $11\frac{1}{9}$
16. 1.07947
17. (a) 1.147 (b) 5.527
18. (a) 2.694492 (b) 35.8488 (c) 73.2
19. \$2.14
20. (a) 16 (b) 400

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